When Music meets Mathematics and Fine Arts:

The case of *L'escalier du diable* by György Ligeti

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ABSTRACT
The composition of *L'escalier du diable*, work extracted from the second volume of Ligeti's *Etudes pour piano* (1988-1994), was influenced by the mathematical theory of fractal geometry and visual illusions of the Dutch artist M. C. Escher. The aim of this paper is to propose a new musical analysis of the Ligeti's piano study which takes in great consideration extra-musical elements.

Keywords
György Ligeti, *L'escalier du diable*, fractal geometry, M. C. Escher, musical analysis.

1. BACKGROUND
In 1982 with the *Trio for violin, horn and piano*, Ligeti who is 59 years old, made an "aesthetic turn" and started to renew his musical language. This turn results from new musical preoccupations (e.g. Sub-Saharan polyrhythms, "lamento" motif) and extra-musical influences (e.g. chaos theory, fractal objects of B. Mandelbrot).

2. METHODOLOGY
Three steps are necessary to present my analysis of *L'escalier du diable*. Thus, I hope that I will respect the thinking and the true motivations of Ligeti.

2.1 Ligeti's statements
Ligeti often expressed himself on his music as many articles and interviews of the composer show. These documents are essential for understanding and rigorously analyzing Ligeti's works. According to my knowledge, neither the composer nor musicologists precisely analyzed *L'escalier du diable*. Nevertheless, Ligeti left us elements and clues which will guide the musical analysis of this study for piano.

2.1.1 About Mathematics
"My evolution in the 1980's, was influenced by […] the new science of geometric fractal configurations". [1]

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As to his *Etudes pour piano* (1985-2001), Ligeti said: "Further influences that enriched me come from the field of geometry ([…] self-similar forms from fractal geometry), whereby I am indebted to Benoît Mandelbrot and Heinz-Otto Peitgen for vital stimulus". [2]

"I am finding a new music turned to fractal objects". [3]

2.1.2 About M. C. Escher's works
"Illusionistic drawings of Maurits Escher more clearly appear in the recent works like the *Etudes pour piano*". [4]

"In 1972, when I could see the drawings of Escher, they influenced me a lot". [5]

2.2 Explanations of extra-musical influences
It is important to clearly study extra-musical influences quoted by Ligeti in order to understand the exact repercussions in his *Etude*.

2.2.1 Fractal geometry
This mathematical domain seeks to reveal the specific internal organization of an object. A fractal object presents a scale invariance implying a recurrence, a motif inside another motif. "L'escalier du diable" ("The devil's staircase") comes from fractal geometry. This staircase presents "an infinite number of steps and sizes […] there exists very small and very large steps". Moreover, the devil's staircase gets "a finite length contrary to all mathematical fractals" [6]. In addition, this staircase presents a horizontal segment in its center, which does not undergo a change in spite of successive iterations coming from mathematical process.

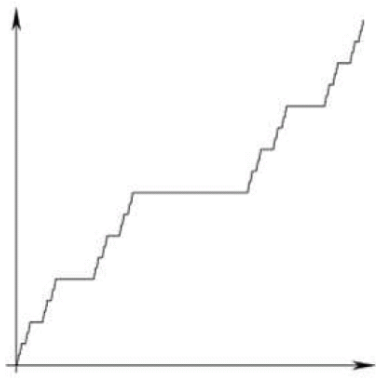

Figure 1. "The devil's staircase".

2.2.2 Escher's staircases

Escher's works fascinate the audience and lead it to differently see things and break with traditional visual marks. This artist had "a passion for the regularity and the mathematical structure, the continuity and the infinity […] and the three dimensional representation in two dimensions" [7]. Escher created visual illusions thanks to different processes. On the one hand, he used "the periodic fillings of plan" which can bring a sensation of infinity and, on the other hand, he used complex perspectives. With the staircase's figure, Escher could apply his ideas about visual illusions. For instance, when we look at *Ascending and descending* and *Relativity* by Escher, we can not affirm if people are descending or ascending the staircases. Moreover, we can remark that the first Escher's lithograph quoted is inspired by an impossible mathematical object: "the Penrose staircase". This staircase presents a visual ambiguity: it seems ascending and descending at the same time according to the direction of rotation selected by the audience.

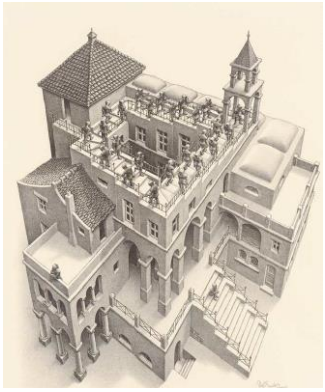

Figure 2. *Ascending and descending* by Escher.

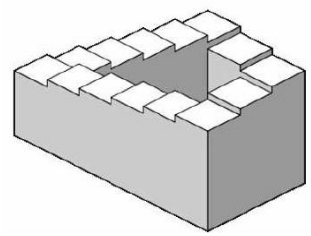

Figure 3. The "Penrose staircase".

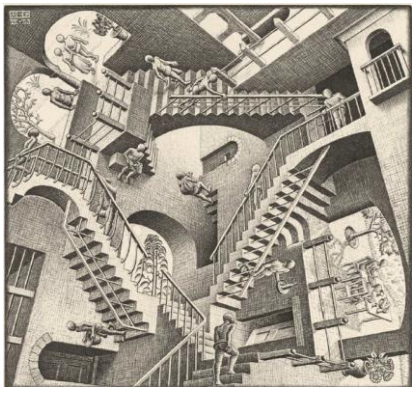

Figure 4. *Relativity* by Escher.

2.3 Musical analysis

We can now study how the composer transposed extra-musical elements in music as the main characteristics and ideas coming from extra-musical influences previously outlined.

The musicologist Richard Steinitz proposed a brief musical analysis [8] based on the fractal geometry but he did not consider formal analogies. The staircase's steps are illustrated by the quasi-uninterrupted repetition of every notes of chromatic scale. Every note is given at rhythmic irregular intervals what reminds the different sizes between the steps of the devil's staircase.

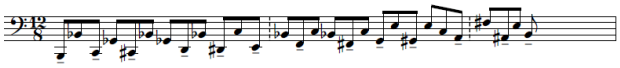

Figure 5. Staircase's steps, excerpt from bar (1).

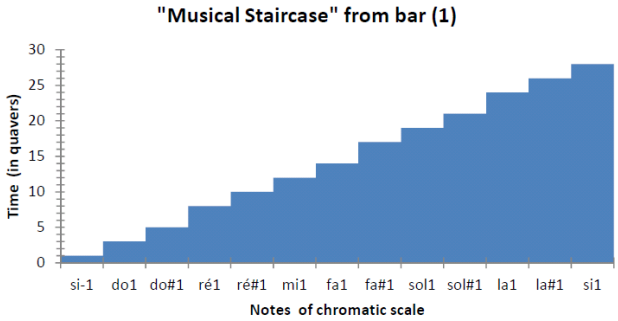

Figure 6. Graphical representation of Figure 5.

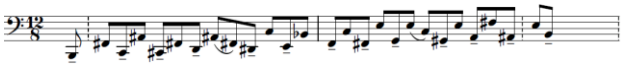

Figure 7. Staircase's steps, excerpt from bars (3) & (4).

Furthermore, the tripartite form of *L'escalier du diable* imitates the three main zones of the mathematical devil's staircase. Indeed, the centre of the *Etude* (bars (26)-(43)) corresponds to a brief lull similar to the horizontal segment of the mathematical staircase. This moment is characterized by succession of longer rhythmic values (e.g. dotted semibreve) contrary to the uninterrupted succession of quavers at the beginning and at the end of the *Etude*. What is more, we can draw a parallel between the idea of the finite length of the devil's staircase and the last bars of *L'escalier du diable* where the two "limits-notes" of the keyboard are emitted (*la*-2 & *do*7).

The *Etude* by Ligeti presents a sound ambiguity. Indeed, even if the composer constantly uses the chromatic ascending scale, the impressions of ascending and descending subsist because the chromatic scale does not always start to the same note and register. Thus, this process is similar to visual illusions in the staircases of Escher where people seem to ascend and descend at the same time. On the other hand, the complex polyrhythmy and polyphony added to notes simultaneously played in the low, medium and high pitched registers (e.g. bar (31): system with 4 staves) create sound perspectives similar to visual perspectives in the works of Escher.

3. CONCLUSION

The extra-musical influences were a profound inspiration for the composer and they played a main role in his musical conception. A musical analysis which considers extra-musical elements is so very important to comprehend a difficult musical composition like the *Etudes* by Ligeti. For instance, the study of the mathematical devil's staircase permitted to understand the musical form used by the composer. Other works by Ligeti, like the first piano etude, *Désordre*, or the fourth movement of *Piano Concerto*, have a formal structure directly inspired by the chaos theory and the fractal geometry ; the "gigantism" of the fourteenth piano etude, *Columna Infinită*, comes from Brancusi's eponymous sculpture. Finally, we become aware of when music meets mathematics and fine arts, the result can be extremely salutary and rewarding for musical analysis.

4. REFERENCES

[1] Ligeti, G. *Notes de programme - Ligeti/Mahler - du mercredi 14 au lundi 26 mai 2003*, Cité de la musique, Paris (2003), 32.

[2] Ligeti, G. booklet in *György Ligeti - Works*, Sony Music, 2010 (9 CD), 41.

[3] Albèra, P. and Ligeti, G. Entretien avec György Ligeti in *Musiques en création*, Genève, Contrechamps, 1997, 85.

[4] Ligeti, G. *Neuf essais sur la musique*, Genève, Contrechamps, 2001, 20.

[5] Ligeti, G. and Michel, P. *György Ligeti : compositeur d'aujourd'hui*, deuxième édition revue et complétée, Montrouge, Minerve, 1995, 193.

[6] Sapoval, B. *Universalités et fractales - Jeux d'enfant ou délits d'initié ?*, Manchecourt, Flammarion, 2001, 209.

[7] Ernst, B. *Le miroir magique de M. C. Escher*, Berlin, Taschen, 1986, 16.

[8] Steinitz, R. Music, math & chaos in *The Musical Times* vol. 137 n°1837 (1996), 14-20.

Steinitz, R. *György Ligeti: Music of the imagination*, London, Faber and Faber, 2003, 307-310.